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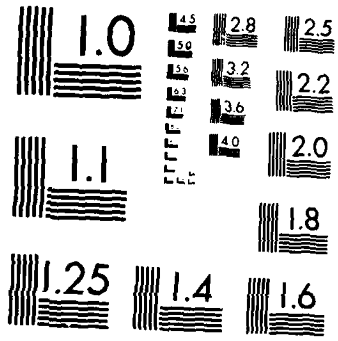
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In the first year of an anticipated three year program the research has aimed at providing computational tools and procedures as the building blocks for a system to permit efficient solution and high resolution capture of flow structure in gasdynamic problems of realistically complex geometries. The research yielded a comparatively simple algebraic procedure for constructing two dimensional geometry fitted base level composite meshes in quadrilateral patches. The method provides complete control of coordinate distribution and gradient on all patch boundaries which may include slope discontinuities. A robust upwind implicit method (CSCM) was the basis to solve the two dimensional pseudo time dependent Euler or compressible Navier-Stokes equations. Research into solution algorithms for that upwind method yielded a more robust diagonally dominant (DDADI) approximate factorization that subsequently led to a family of rapidly convergent and data storage and management efficient relaxation

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19. Abstract cont.

schemes. Those effectively explicit and unconditionally stable upwind algorithms have led to a simple robust boundary procedure based on interpolation of conservative variable data from other patches overlying interior patch boundaries where coordinates are discontinuous. Results of preliminary tests with model problems show the desired accuracy and great potential for enhancing engineering productivity.

Euler equations, Flow fields,

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Closing Developments in Aerodynamic Simulation with Disjoint Patched Meshes

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Principal Investigator

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Closing Developments in Aerodynamic Simulation
with Disjoint Patched Meshes

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PEDA Corporation

1. Introduction

Recent and projected developments in supercomputers, numerical grid generation techniques and computational algorithms for the compressible Euler and Navier-Stokes equations portend a major revolution in the manner, pace, and cost of design and the resulting performance of aerodynamic systems. To realize these potential benefits, certain closing developments in computational technique must be made in order to effect a highly accurate, reliable, efficient and productive simulation environment for aerodynamic design analysis.

A primary need of the developments is to achieve the capability for a user to easily and rapidly perform flowfield calculations accurately among problems of disparate and realistically complex geometries. The natural approach to realizing this objective with comparatively straightforward extensions of existing computational technology is through the use of systems of quadrilateral patched meshes.

Such systems can be either/both composite (joined) or overset (disjoint). In the former case adjacent patches share a common boundary, or at least parallel boundaries in the case of mesh patch overlap for purposes of applying numerical boundary conditions. With composite meshes, patch boundaries are piecewise fitted to segments of physical or computational boundaries or embedded flow structures. As shown by Lombard, et al[1], composite mesh systems, that may have numerically useful properties of geometric continuity across patch boundaries, admit topologically singular global meshes that have the capability to connect computational regions of great (really any) geometric complexity. However, situations exist where multiple mesh topologies, each naturally related to some different piece of geome-

try or flow structure, offer greater flexibility and accuracy than composite meshing alone. Examples involve multiple bodies that may have relative motion or weak shocks that bear little geometric relationship to boundaries of the flow domain. In such cases systems of disparately oriented and at least partially overset grids as proposed by Berger and Oliger[2] allow arbitrarily high resolution of all features of the flowfield.

To make efficient, productive use of patched meshing strategies requires a body of new computational tools and methodology that are the objectives of the present research. The needed factors are: (1) a simple procedure for generating patched computational meshes with freedom of point and gradient specification on all patch boundaries; (2) improved upwind algorithm/numerical boundary condition procedures for semi-autonomous implicit but unconditionally stable conservative coupling of solutions on a system of multiple patched meshes; and (3) computer graphics, particularly a simple algorithm for constructing contour plots on systems of overset patched meshes. The glue that is to tie these tools together in a simulation requiring minimum human intervention to adapt to new configurations is a flexible parameter controlled multiple mesh data structure. An important objective of the program is to test the evolving techniques in appropriate problems.

2. Research Accomplishments

In the first year of an anticipated three year program the emphasis was placed on the most crucial and challenging objectives – patched grid generation and robust upwind algorithm/boundary procedures for rapid relaxation on multiple meshes.

Algebraic Grid Generation

The concept of patched meshing in which complex domains are broken up into many geometrically regular and topologically rectangular subdomains leads naturally to the use of efficient algebraic techniques for the construction of the individual mesh patches. To obtain the desired smoothness properties over the global mesh in the vicinity of patch boundaries, a technique that permits specification of point

distribution and gradient on all boundaries was devised. The technique – termed generalized transfinite interpolation[3] – makes use of a parameterized general cubic polynomial for the coordinate curves. Regularity of the mesh is obtained by employing continuous distributions of the parameters of the curves within judiciously chosen bounds based on analysis. Stretching functions such as that of Vinokur[4] are used to distribute points and blending functions are used to distribute parameters of the curves between lateral boundaries.

A novel feature of the technique is the introduction of the corner singularity from analysis to govern distribution of points and parameters in the vicinity of boundary slope singularities. At such points, the method thus obtains the desired properties of mesh smoothness to the interior. A global mesh solution obtained with the method for a backward step problem is shown in Figure 1. Here the solution was generated in two patches one containing the exterior corner and the other the interior corner. The solution was matched analytically at the patch interface. Additional background, details of the technique and other numerical results are contained in reference 3.

Upwind Implicit Relaxation Algorithm/Boundary Procedures

Under the contract we have devised a new single level operationally explicit but effectively implicit algorithm for gasdynamics. The algorithm is particularly appropriate for multiple patch mesh systems because each solution sweep operation on any patch is decoupled from any other. Thus the method is not only very storage efficient and simple to program including coupling at patch boundaries, but also can make excellent use of parallel computing in several straightforward ways.

Previously the Beam-Warming factored implicit algorithm[5] with the Baldwin-Lomax thin layer viscous approximation[6] has provided the basis for two similar space marching (PNS) procedures[7,8] for the compressible Navier-Stokes equations. These PNS methods which are highly efficient – requiring half the data storage and a small fraction of the computer time of two level time dependent methods –

have proven effective for flows[9,10] with favorable streamwise pressure gradient or with relatively small adverse pressure gradients. However, in the presence of strong adverse pressure gradient such as occurs in a wing or fin root regions the contemporary PNS methods suffer numerical stability problems and may infer streamwise separation even where separation doesn't occur[11]. In such unseparated (perhaps weakly separated) regions, numerical stability has been maintained at the price of employing large amounts of artificial viscosity with a resulting loss in predictive accuracy and knowledge of the actual state of the flow. Where strong streamwise separation occurs the methods are unstable and cannot proceed. Particularly for the increasingly relevant laminar flow situation that will be encountered at very high altitude by aerodynamic systems such as orbital transfer vehicles (OTV's) and space shuttle, streamwise separation becomes a likely occurrence[12] in compression corners associated with canopies, pods, flared bodies, wing or fin roots and deflected control surfaces. Thus a more general technique is needed that is inherently stable for all types of upstream influence. At a minimum the mixed elliptic hyperbolic problem requires global iteration, preferably with type dependent differencing. Various steps in this direction have recently been taken by Rakich[13], by Rubin and Reddy[14] and by Rizk and Chaussee[15].

Rakich[13] utilized global iteration with Vigneron's Mach number dependent upwind and downwind splitting[8] of the pressure gradient for the streamwise momentum equation. The approach provides an improvement in both accuracy and stability for both weakly interactive boundary layer flow and strongly interactive flow without (significant) streamwise separation. Global iteration is carried out by marching the PNS equation repeatedly only along the downstream direction.

For incompressible flow Rubin and Reddy[14] (also Lin and Rubin[16] for supersonic flow) introduced type dependent (upwind) differencing of the streamwise velocity along with pressure splitting in an implicit method involving a staggard grid - dependent variable location scheme. This method was extended to compressible

flow by Khosla and Lai[17]. The method admits strong interaction with streamwise separation but is not homogeneous across the spectrum of Mach number regimes of interest here. Other PNS related methods for subsonic flow and boundary layer flow with streamwise reversal have recently been reviewed by Brown[18] in conjunction with a new staggered grid scheme.

In an attempted generalization, Rizk and Chaussee[15] presented a hybrid technique of space marching in supersonic zones and relaxation with a Beam-Warming time dependent central difference method over zones of strong upstream influence with streamwise separation. They presented two variations of the time dependent method: one fully implicit requiring two levels of storage and matrix inversion procedures in all space directions, the other explicit in all space directions but the thin layer direction and requiring only one level of storage as in a space marching algorithm. Both procedures require the same several hundred iterations minimum to reasonably converge, with the semi explicit procedure requiring substantially less machine time. However, both procedures are inherently unstable, except for the use of artificial dissipation. The marginal stability coupled with the lack of type dependent differencing and well posed boundary approximations all contribute to slow convergence. In balance, while workable, the hybrid approach leaves something to be desired from the points of view of convenience, computer time and internal consistency of the global solution procedure.

A new globally iterated scheme related to that which will be presented here, has been presented by Moretti[19]. In his procedure for steady inviscid flows, the Euler equations are cast in Riemann variables and the resulting uncoupled equations are solved by integrating each Riemann variable separately, sweeping back and forth alternatively along each coordinate line. The coupling between the equations and, thus, the non-linearity are introduced only through the boundaries and the updating of state after complete sweeps. As can be inferred from the results to be presented here, the reduced coupling inherently limits the rate of convergence of Moretti's

method and the procedure is not extendable to the compressible Navier-Stokes equations.

New Universal Single Level Scheme CSCM-S

The CSCM flux difference eigenvector split upwind implicit method[20,21,22] for the inviscid terms of the compressible Navier-Stokes equations provides the natural basis for an unconditionally stable space marching technique through regions of subsonic and streamwise separated flow. In such regions the split method can be likened to stable marching of each scalar characteristic wave system in the direction of its associated eigenvalue (simple wave velocity). In supersonic flow, where all eigenvalues have the same sign, the method automatically becomes similar to the referenced PNS techniques based on the Beam-Warming factored implicit method with the Baldwin-Lomax thin layer viscous approximation.

Compared to contemporary central difference methods, the CSCM characteristics based upwind difference approximation with its inherent numerical stability leads to greatly reduced oscillation and greater accuracy in the presence of captured discontinuities such as shocks, contacts and physical or computational boundaries. The correct mathematical domains of dependence that correspond with physical directions of wave propagation are coupled with well posed characteristic boundary approximations[21] naturally consistent with the interior point scheme. The result is faster sorting out of transient disturbances and substantially more rapid convergence to the steady state. The splitting and the associated time dependent implicit method have been described in detail in references (20) and (22) for quasi 1-D and 2-D planar or axisymmetric flow.

In the following, we will sketch the differences between the time dependent method and the new space marching technique which we designate CSCM-S. The discussion will begin with the quasi 1-D inviscid formulation, present some results elucidating the properties and performance of the method, then give additional details entering into multidimensional inviscid and thin layer viscous procedures

and, lastly, present early 2-D solutions obtained with the new single level scheme in problems solved previously[22] with the time dependent method.

Quasi 1-D Formulation

The general jth interior point difference equations for the time dependent CSCM upwind implicit method is written

$$(I + \widetilde{A}^{+}\nabla + \widetilde{A}^{-}\Delta)\delta q_j = -\widetilde{A}^{+}\Delta q)_{j-1}^{n} - \widetilde{A}^{-}\Delta q)_{j}^{n} \tag{1}$$

where ∇ and Δ are backward and forward spatial difference operators. In the notation the interval averaged matrices between node points j and $j+1$ are labeled j. The right hand side of equation (1) is written for the first order method. Higher order methods in space are given with results in references 20 and 22.

Central to its accurate shock capturing capability, the CSCM conservative flux difference splitting has the "property U" put forth by Roe[23]

$$(\widetilde{A}^{+} + \widetilde{A}^{-})\Delta q)_j \equiv \Delta F)_j = F_{j+1} - F_j \tag{2}$$

Here q is the conservative dependent variable vector and F is the associated flux vector. The matrices $\widetilde{A}^{+}$ and $\widetilde{A}^{-}$ are the splittings of the CSCM interval averaged Jacobian matrix according to the signs of the averaged eigenvalues. Thus in the equation for the jth grid point, $\widetilde{A}^{+}\Delta_{\xi}q)_{j-1}$ represents stable characteristic spatial differencing backward for positive eigenvalue contributions and $\widetilde{A}^{-}\Delta_{\xi}q)_j$, forward for negative ones.

With $\delta q = q^{n+1} - q^{n}$, equation (1) defines a two level linearized coupled block matrix implicit scheme that can be solved by a block tridiagonal procedure. In reference (22) a new (DDADI) approximately factored alternating sweep bidiagonal solution procedure for equation (1) is presented that is shown to be very robust and is effectively explicit, i.e. requires only a decoupled sequence of local block matrix inversions rather than the solution of the coupled set. For the forward sweep the bidiagonal solution procedure can be written

$$(I + \widetilde{A}^{+} - \widetilde{A}^{-})\delta q^{*}{}_{j} = RHS + \widetilde{A}^{+}\delta q^{*}{}_{j-1} \tag{3}$$

For the linear problem, i.e. constant coefficient case of stability analysis, equation (3) is equivalent to the single level space marching procedure

$$(I+\widetilde{A}^{+}-\widetilde{A}^{-})\delta q^{*}{}_{j}=\widetilde{A}^{+}q^{*}{}_{j-1}-\widetilde{A}^{+}q^{n}_{j}-\widetilde{A}^{-}\Delta q)^{n}_{j} \tag{4}$$

Nonlinearity enters in the single level space marching form (4) in that at each step of the forward sweep the matrices $\widetilde{A}^{+}$ are averaged between $q^{*}{}_{j-1}$ and q^{n}_{j} rather than homogeneously at the old iteration level n. Similarly, a companion backward space marching sweep that is symmetric to equation (4) and that is intimately related to the backward sweep of the alternating bidiagonal algorithm of reference (22) is

$$(I+\widetilde{A}^{+}-\widetilde{A}^{-})\delta q_{j}=-\widetilde{A}^{+}\Delta q^{*})_{j-1}+\widetilde{A}^{-}q^{*}_{j}-\widetilde{A}^{-}q^{n+1}_{j+1} \tag{5}$$

The method given by equations (4) and (5) is von Neumann unconditionally stable for the scalar wave equation. The analysis shows the significance of DDADI approximate factorization in rendering both the forward and backward sweeps separately stable regardless of eigenvalue sign. Consequently as the local Courant number becomes very large, the robust method becomes a very effective (symmetric Gauss-Seidel) relaxation scheme for the steady equations, a fact which substantially contributes to the very fast performance that will be demonstrated.

At a right computational boundary on the forward sweep we solve the characteristic boundary point approximation[22]

$$(\widetilde{\widetilde{A}}^{+}+\widetilde{A}^{+})\delta q^{*}_{N}=\widetilde{A}^{+}q^{*}_{N-1}-\widetilde{A}^{+}q^{n}_{N} \tag{6}$$

$q^{n+1}_{N}=q^{*}_{N}$ and at a left, on the backward sweep

$$(\widetilde{\widetilde{A}}^{-}-\widetilde{A}^{-})\delta q_{1}=\widetilde{A}^{-}q^{n}_{1}-\widetilde{A}^{-}q^{n+1}_{2} \tag{7}$$

Following the solution of equations (6) and (7) the conservative state vector is iteratively corrected[21] to maintain the accuracy of prescribed boundary conditions while not disrupting the representation of the computed characteristic variables

running to the boundary from the interior. Analysis of a model system with upwind differenced scalar equations and coupled boundary conditions was related to the linearized bidiagonal scheme[22] by Oliger and Lombard[25]; the analysis also strongly supports the numerically confirmed robust stability of the present nonlinear method for gasdynamics. In contract to reference 22, a useful result of reference 25 is that on the forward sweep there is no need for a predictor step at the left boundary $J = 1$; to the solution sweep begins at $J = 2$. Similarly, the backward sweep begins at $J = N - 1$.

With the updating at each step, where in equation (5) $\delta q_j = q_j^{n+1} - q_j^*$, it is clear that the symmetric pair of equations (4) and (5) serve to advance the solution two pseudo time (iteration) levels; whereas, the linear alternating bidiagonal sweep algorithm of reference (22) advances the solution only one level. To maintain conservation to a very high degree, in single sweep marching in supersonic zones we iterate (at least) once locally at each space marching step. The local iteration serves to make the eigenvectors in the coefficient matrices consistent with the advanced state and, thus, provides improved accuracy for the nonlinear system. It appears effective to do this inner iteration everywhere, i.e. in both subsonic and supersonic regions, as the number of global iteration steps to convergence with two inner iterations has been found reduced by a factor of three to four. Since the computational work per two steps is about the same for the single level and two level schemes and beyond the fact that one saves a level of storage in the space marching algorithm, the question arises: Can one get solutions in less computational work through faster convergence with the nonlinear space marching algorithm?

One Dimensional Results

First, we present results for supersonic flow with no shock in Shubin's diverging nozzle. In purely supersonic zones, the experience with the present method is that the solution can be marched accurately in one global iteration, as ought to be the case. Figure 2 shows the exact solution (in solid line) and the computed result

from the first forward sweep. It is evident that the method correctly predicts the solution to plotting accuracy in one forward sweep. With subsequent sweeps the error (the difference between the exact and the computed solution) reduces to machine accuracy in less than three global iterations. In fact, by increasing (from two) the number of inner iterations on the solution procedure at each space marching step, convergence to prescribed accuracy can be guaranteed in one forward sweep. This is also true of contemporary locally linearized unsplit methods in supersonic flow.

With the globally iterative nonlinear space marching formulation, early experience in two quasi 1-D nozzle problems with mixed supersonic-subsonic zones is that solutions are obtained in roughly an order of magnitude fewer iteration steps than had been required with the previously fast pseudo time dependent technique and block tridiagonal solving.

The two nozzle problems which are described and solved by Yee, Beam and Warming[26] and solved with the CSCM time dependent technique in references (20) and (21) are Shubin's diverging nozzle flow and Blottner's converging-diverging nozzle flow. Both problems involve unmatched overpressures at the outflow which result in internal shock terminated supersonic zones and subsonic outflow. For the experiments involving flow of mixed type the same initial data given by Yee, Beam and Warming – a linear interpolation between inflow and outflow values for effectively exact solutions of the problems – is used that was used previously with the time dependent approach.

For flows of mixed type, in Figures 3 and 4 respectively, results are shown for successive forward and backward sweeps for five global iteration steps with Shubin's and Blottner's nozzle flows. In both cases, the exact solution as given by Yee, Beam and Warming is shown in solid line and the present computational results solved on a 51 point mesh, in boxes. Blottner's nozzle flow is shown converged after 10 global iteration steps. There is substantial evidence in other results not shown that

with further work the number of global iterations required to compute flows such as Blottner's can be reduced by a factor of two, to about five.

In Figure 5, we show a subcritical, i.e. completely subsonic, flow solution computed in only two global iteration steps for the Blottner nozzle geometry with different inflow conditions. Here the exact analytical solution derived by Venkatapathy is shown in solid line and our computed results in boxes.

The alternating direction sweeps in our method have been derived directly out of theory for solving the implicit set of difference equations. However, one can see mechanistically, numerically speaking, that omitting the backward sweep from the pair and globally iterating only with the forward sweep equation (4) will result in permitting the influence of a subsonic outflow boundary (or interior disturbance) to propagate upstream only one grid point per global iteration. In such a case, which relates to other global iteration methods found in the literature and that also sweep only in the main flow direction, the rate of convergence is greatly inhibited relative to symmetric sweeping by a factor of order roughly the number of grid points in the subsonic zone. Mathematically, this inhibition is the result of the failure to include in the implicit process the effect of the eigenvectors governing upstream influence but to treat these waves explicitly with effective CFL unity.

In Figure 6 we illustrate the progress of the transient solution to the subcritical nozzle problem after 15 forward sweeps, with the backward sweeps omitted. One can clearly see that the wave influence of the outflow boundary has progressed only 15 mesh points forward of the outflow boundary. In Figure 7 the transient solution is shown after 60 steps which is beyond one characteristic transit time (equivalent to 50 mesh intervals) for the upwind wave to reach the inflow boundary. In Figure 8 we show the history of the RMS error in the primitive variables. The solution is found to converge to roughly the same RMS error after three characteristic times (150 steps) as the solution obtained with the symmetric alternating sweep sequence after only 3 global iterations.

Blottner's supercritical nozzle problem which involves subsonic inflow accelerating through a sonic point to a supersonic zone terminated by a shock to subsonic outflow is the most computationally demanding of the test cases and indicates the capability for the method to compute simply and consistently over the subsonic forebody and base regions of blunt bodies in supersonic flow. Thus the need for separate time dependent codes will be obviated by this new method.

Finally, in Figures 9 and 10, we present the convergence history for the present nonlinear scheme and the linearized time dependent scheme for completely subsonic and supersonic nozzle flows. The x-axis shows the number of iterations each scheme requires to reduce the exact error to five orders of magnitude for various Courant numbers. It is evident that the present scheme converges extremely fast at all CFL numbers compared with the method based on the linearized block tridiagonal solver.

Two Dimensional Formulation

For two dimensional flow, assuming a marching coordinate ξ, inviscid terms

$$B^{+}\nabla_{\eta} + B^{-}\Delta_{\eta} \tag{8a}$$

and

$$-B^{+}\Delta_{\eta}q)_{k-1} - B^{-}\Delta_{\eta}q)_{k} \tag{8b}$$

are added to the left and right hand sides respectively of both the forward and backward sweep equations (4) and (5). For viscous flow, second centrally differenced, thin layer viscous terms are also added in the η direction as is conventionally practiced, e.g. Steger[27]. With the terms for the η cross marching coordinate direction, the technique now becomes an implicit method of lines. Along each η coordinate line, one can solve the equations coupled with a block tridiagonal procedure. Alternatively, a further DDADI bidiagonal approximate factorization can be employed in the η direction and solved either linearly as in reference (22) or nonlinearly as here in the ξ direction. As shown in the quasi 2-D numerical experiments of reference (22), DDADI bidiagonal approximate factorization is stable for viscous as

well as inviscid terms. Finally in reference (22) there is a relevant discussion of the reduced approximate factorization error that attends using DDADI in one or more space directions. A variety of multidimensional solution strategies derived from DDADI bidiagonal approximate factorization in a simple symbolic algebra based on the difference stencil are presented in reference 24.

While extensions to 3-D are not given in detail here, we note such extensions to a developing two level 3-D CSCM upwind scheme are possible and will be adopted in the future. For 3-D, the inviscid terms similar to (8a) and (8b) will again be added for the ς cross flow direction. For efficiency in solving each resulting marching plane, the implicit operators for the η and ς coordinate directions can be approximately factored using DDADI as described in reference (22) for two space directions.

Two Dimensional Results

We present results for a 45° - 15° axisymmetric transonic nozzle flow previously studied experimentally by Cuffel, Back and Massier[28] and computationally by Cline, Prozan, Serra and Shelton (all referenced in (28)) and ourselves[22]. In Figure 11 we show results after 10 steps of an early computation run at a local CFL number of 20 with the present first order single level scheme. Except for the addition of an error correction procedure[21] to counter numerical inflow boundary condition drift, a factor which has improved the present solution in the vicinity of the axis, the effectively converged results found here are the same as those given for the two level scheme in reference 22. (As long as the problem has a unique solution, the two schemes must give equivalent results since the right hand side difference equation sets, including boundary approximations, are the same.)

For the solution given in Figure 11, we noted a very rapid rate of reduction in residual, three orders of magnitude in ten steps. This compares with 60 steps given in reference (22) for the solution obtained with the two level scheme. The rapid convergence found in this transonic problem for the CSCM-S method with viscous terms provides the reasonable expectation of similar fast results to be ob-

tained without viscous effects. Thus the method in multidimensions appears to have attractive potential for an improved transonic Euler solver as well as Navier-Stokes solver.

Next, we present first order inviscid and viscous results for an inlet problem shown in Figure 12. The pressure contours for the first order inviscid solutions are shown in Figure 13. Figure 14 shows the first order viscous results. The viscous computation shows the presence of the leading edge shock. The flow structure compares very well with the theoretical (for the inviscid case) and other computational results. In Figure 15, the inviscid and viscous wall pressure are compared with the exact solution (inviscid). Figure 16 shows the convergence history of the RMS residue of all the conservative flow variables for the inviscid problem solved at CFL = 100 with 4 inner iterations at every axial location. For the inviscid case, only forward marching was carried out and backward marching was omitted. The solution has converged for practical purposes at the end of the first sweep. The residue reaches machine accuracy in 10 iterations. In a later paper[29], we show the residual reduction versus inner iteration number in single sweep solutions for supersonic flow and compare results with contemporary PNS procedures.

Patched Mesh Boundary Procedures

In previous work Lombard, et al[21,22] and Oliger and Lombard[25] gave stable implicit procedures for computing the solution at external boundaries of a computational domain. Those procedures generalized the work of Kenzer to matrix coupled linearized boundary conditions complementing a set of advective difference equations (associated with well posed characteristics) to the boundary.

Under the present contract we have explored the problem of implicitly coupling at interior patch boundaries the global solution on a system of multiple patch meshes. The approach we have taken in this research is numerical experimentation among a number of boundary treatment approximations to the solution of the continuous domain problem. For comparison purposes with previous single mesh

results, numerical experiments were performed with the well tested two level pseudo time relaxation CSCM scheme at the interior points of the mesh patches.

Our first experiments, to be described here, dealt with breaking a single coordinate line into segments and solving sequentially on each the equations for a quasi 2-D viscous compressible flow. The model problem, with which we experimented in reference 22 for an uninterrupted mesh, is a transient pipe flow resulting from an initially nonequilibrated pressure between the axis and wall boundaries. Three kinds of cases were run with the two-level linearized implicit procedure; all featured sequential solving on patches with at least one point of overlap. Case 1 had frozen boundary data, obtained from the solution on neighboring meshes in an operationally explicit manner. Specifically at left and right first computed interior points of a patch, we solved the bidiagonal equations respectively

$$(I + \widetilde{A}^{+} - \widetilde{A}^{-}\Delta)\delta q_2 = \widetilde{A}^{+} q_1^{n} - (\widetilde{A}^{+} - \widetilde{A}^{-}) q_2^{n} - \widetilde{A}^{-} q_3^{n} \tag{9a}$$

and

$$(I - \widetilde{A}^{-} + \widetilde{A}^{+}\nabla)\delta q_{N-1} = \widetilde{A}^{+} q_{N-2}^{n} - (\widetilde{A}^{+} - \widetilde{A}^{-}) q_{N-1}^{n} - \widetilde{A}^{-} q_{N}^{n} \tag{9b}$$

Here the symbols $n, n+1$ indicate the "frozen" boundary data to come from the solution on the interior of an adjoining and partially overlying patch may be either at the old or new iteration level in the global procedure. In Case 1 the solution on all the grids was effectively updated at the same time.

Case 2 featured reverse sequential cycling of the two level time dependent solution procedure through the grids on alternate global steps. In the forward sequence the right interior patch boundaries were (a) frozen or (b) computed with one-sided characteristic boundary conditions obviating any change in the characteristic data from outside (to the right of) a patch. The left interior patch boundaries inherited implicit data (δq) from the solution on the computed patch to the left. In the backward sequence all the roles were reversed including the directions of forward elimination and back substitution in the tridiagonal matrix inversion procedure.

Case 3 featured cycling through the patches in the predictor (forward elimination) step of the solution procedure, inheriting implicit left boundary data as in Case 2. Then the patches were cycled through in reverse order on the corrector (back substitution) step. The result (save for interpolation if data points of the grids were interlaced) is identically equivalent to solving on an uninterrupted single patch mesh.

The results of the three sets of experiments were comparable for local time steps based on constant CFL number up to about 50. Beyond that the rate of convergence of the solutions for Cases 1 and 2 diminished and at sufficiently high CFL number failed to converge. The effort involved in Case 2 with the two level scheme was not rewarded relative to the simple procedure of Case 1. Within the framework of the single-level symmetric Gauss-Seidel implicit relaxation scheme, however, Case 2a is operationally no more difficult than Case 1 and more closely approximates Case 3. Case 2b has a consistency problem that inhibits firm convergence.

Figure 17 compares rms residual (density) convergence history for Cases 1 and 3 with three mesh segments with two cells of overlap and run at CFL 25. As one might expect, the effectively uninterrupted mesh procedure is found to converge (two to three times) faster for about the first 50 steps through the major transient. After that the performance is comparable. The performance of the frozen boundary treatment Case 1 with five segments at CFL 25 was found to be not materially worse than with three segments, but the performance degradation with increasing CFL was found to proceed faster with increasing number of patches.

From the results of the quasi 2-D experiments and extrapolation from experience with the single level scheme, we observe that the simple boundary procedure with frozen conservative variable data taken from the solutions on adjacent meshes and coupled implicitly by alternating direction sequential solving through the patches has robust stability to sufficiently high CFL number to yield a rate of convergence meeting our needs.

3. Professional Personnel

Professional researchers who contributed to this project are

Dr. Charles K. Lombard, Principal Investigator

Professor Joseph Oliger, Consultant

Dr. Marcel Vinokur, Consultant

Dr. Ethiraj Venkatapathy

Dr. Jorge Bardina.

4. Interactions

The research described in this report has to this point been partially presented in the form of a paper on algebraic grid generation by Vinokur and Lombard (reference 3), a SIAM meeting paper (reference 25) by Oliger and Lombard on boundary procedures for bidiagonal alternating sweep schemes and a Computational Fluid Dynamics Seminar at NASA-Ames Research Center by Lombard on the Universal Single Level Implicit Algorithm. The latter invoked tremendous interest and discussion.

The research is about to spawn three other papers on the single level relaxation algorithm – references 24 and 29 and the unreferenced paper

Lombard, C.K., Venkatapathy, E. and Bardina, J.: Forebody and Baseflow of a Dragbrake OTV by an Extremely Fast Single Level Implicit Algorithm, AIAA-84-1699, Snowmass, Colorado, June 1984.

5. New Discoveries

The Universal Single Level Algorithm for the compressible Euler or Navier-Stokes equations is a new discovery in numerical methods that promises to result in substantial efficiences in data storage, programming, machine time and human productivity.

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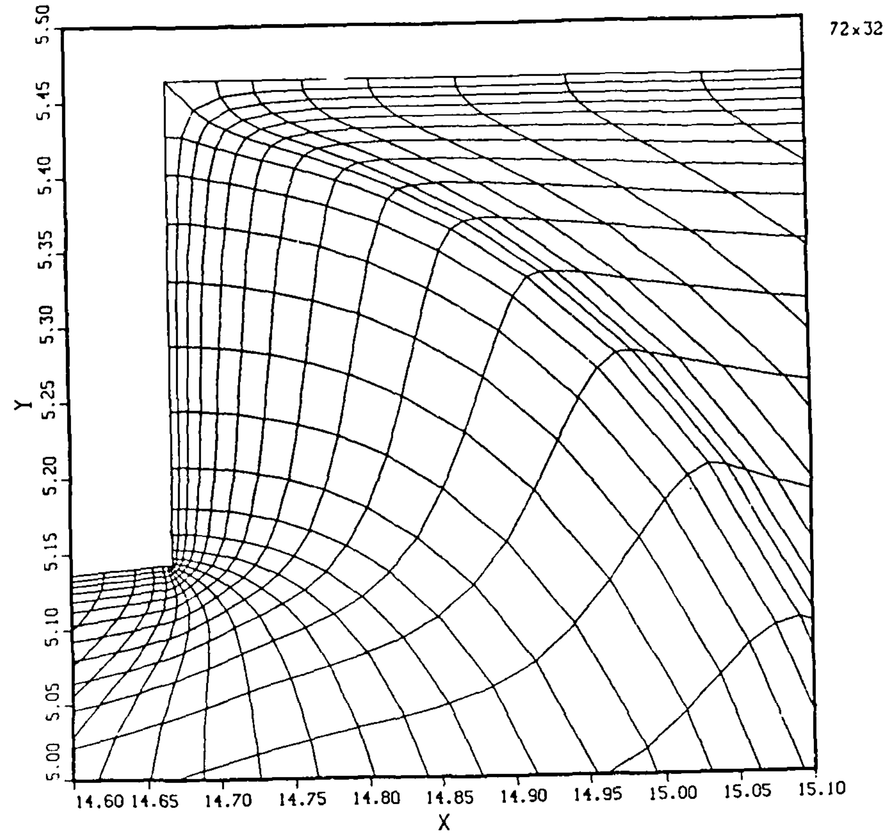


Figure 1. Portion of an algebraically generated computational mesh for a flow domain containing an external and an internal corner.

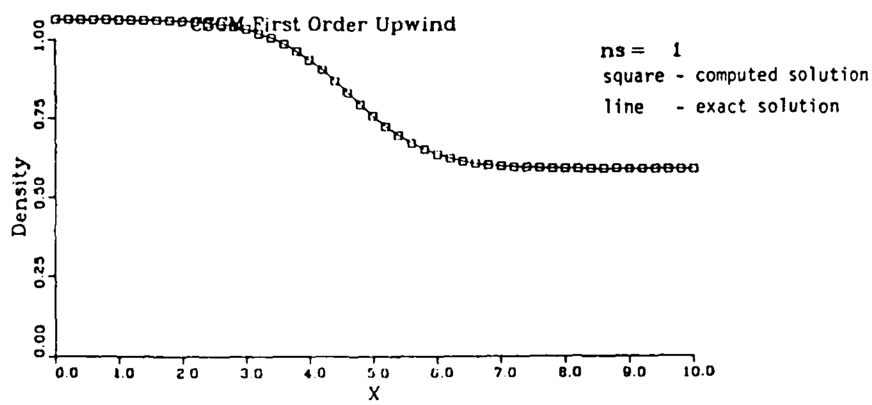


Figure 2. Shubin's diverging nozzle supersonic flow solution developed in one forward sweep from supersonic initial data.

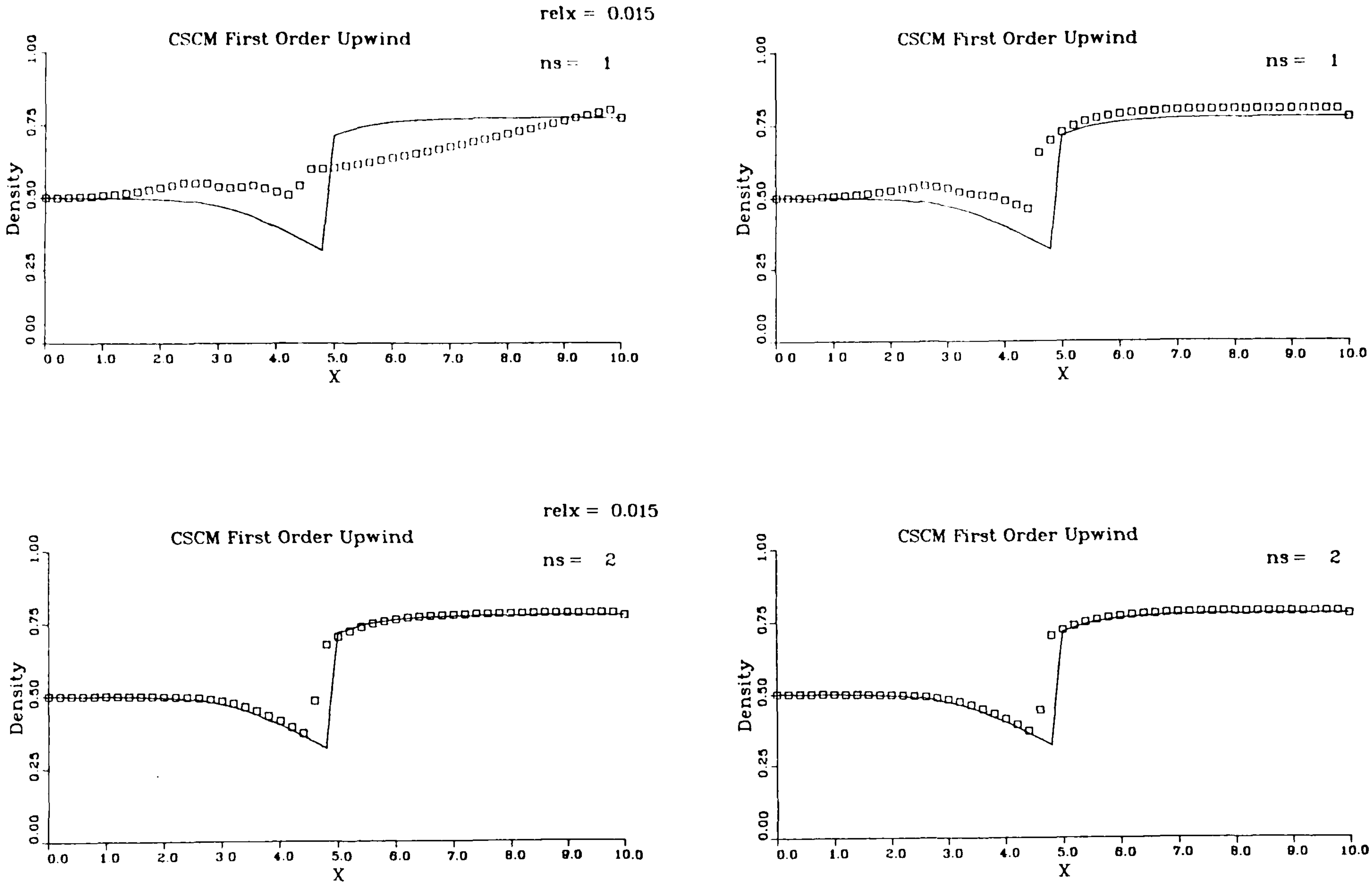


Figure 3. Shubin's diverging no;zle flow solution developed in alternating forward and backward sweeps, one each per global iteration step for five steps. square - computed results , line - exact solution

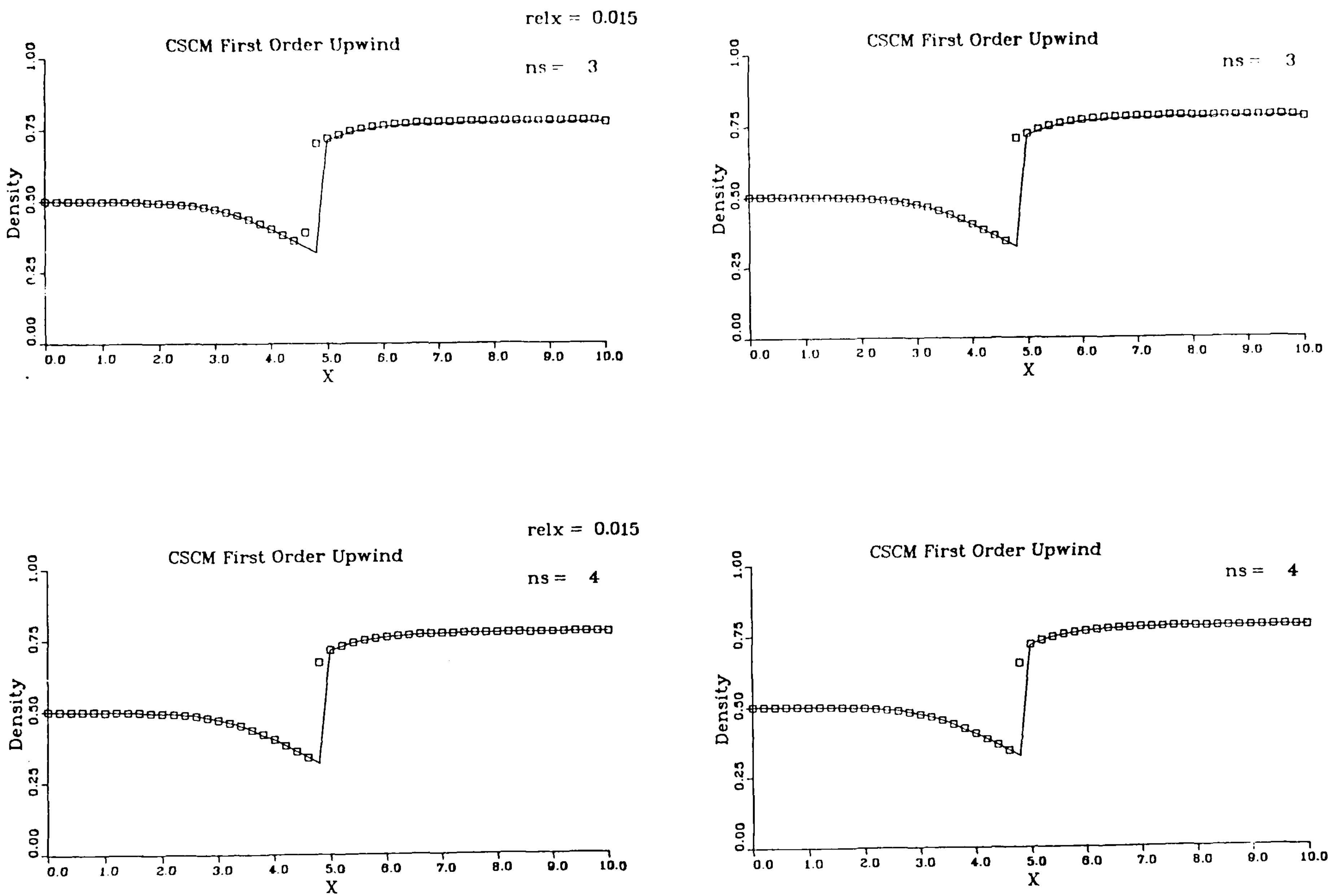
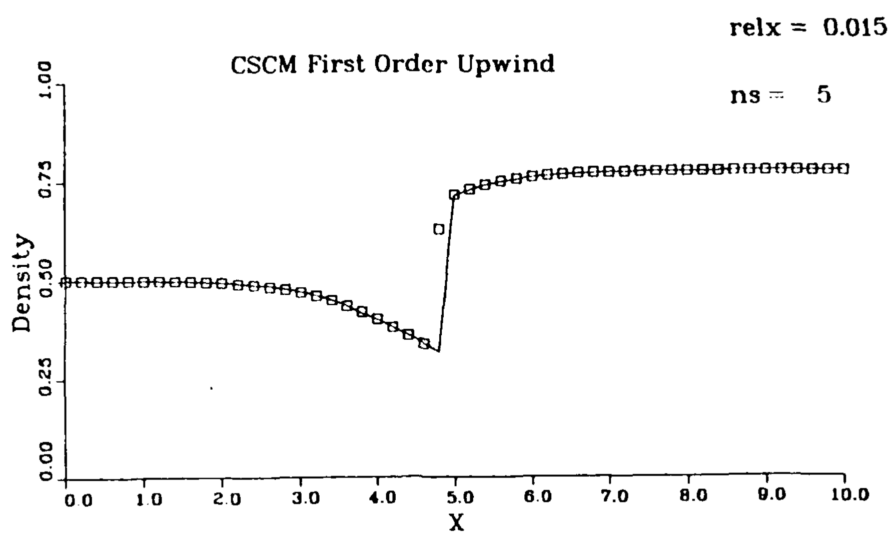


Figure 3. continued



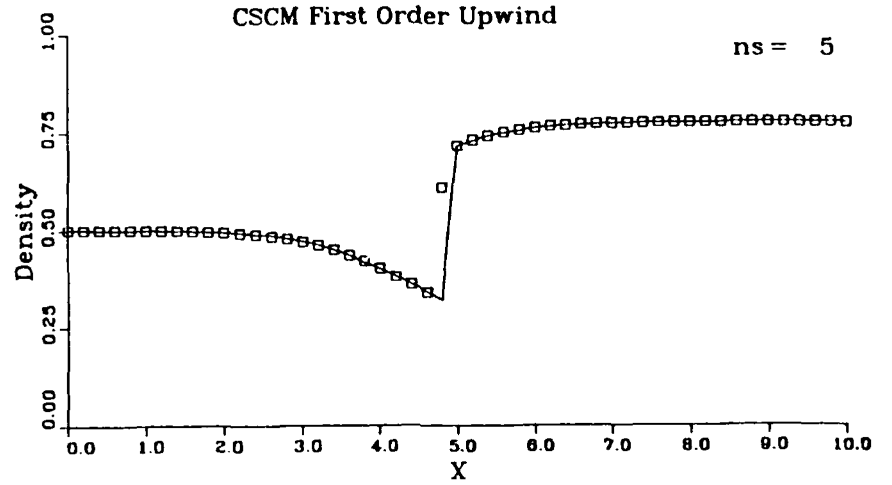


Figure 3. continued

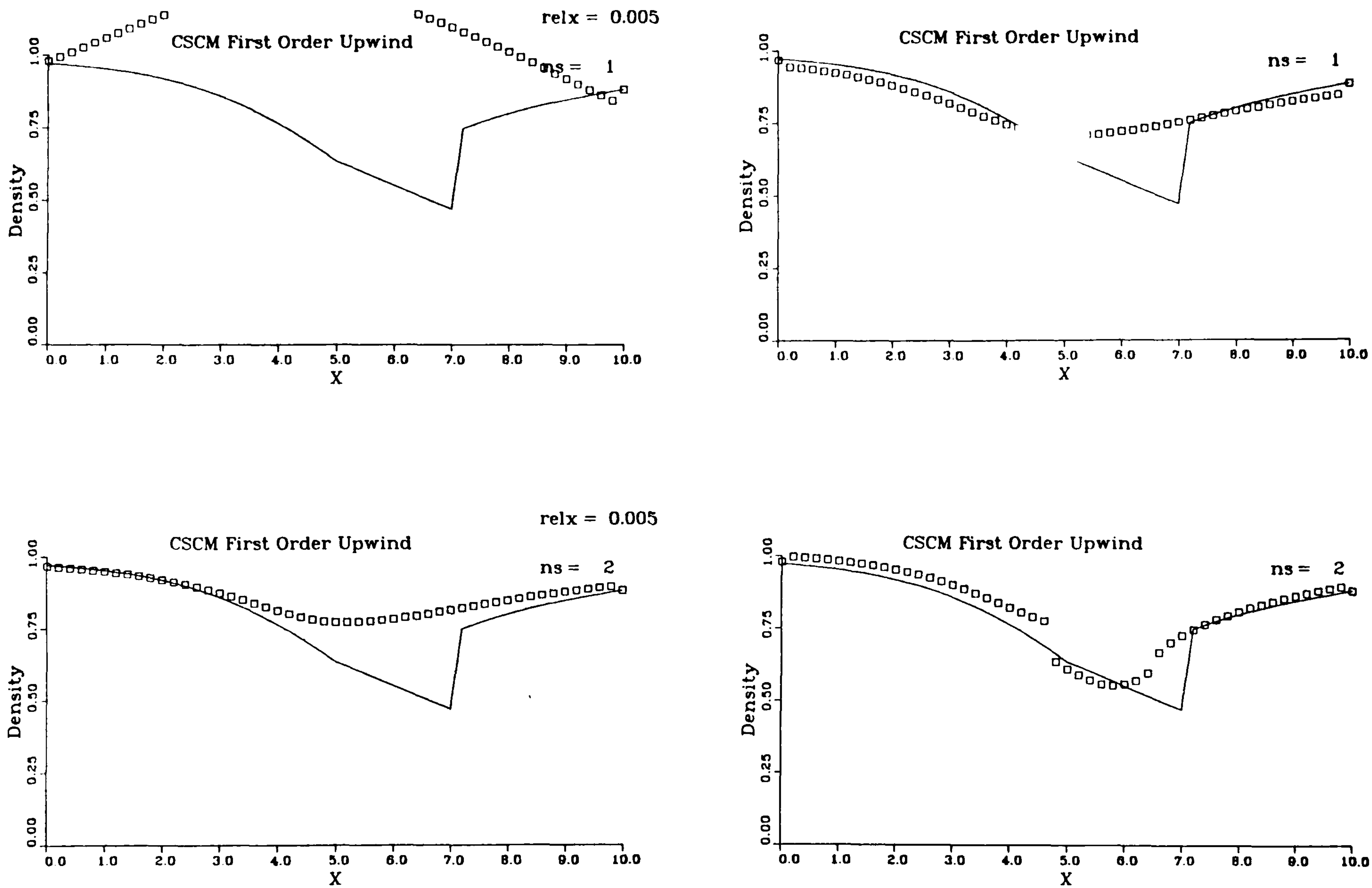


Figure 4. Blottner's converging-diverging supercritical nozzle flow solution developed in alternating direction sweeps for ten global iteration steps. square - computed results , line - exact solution

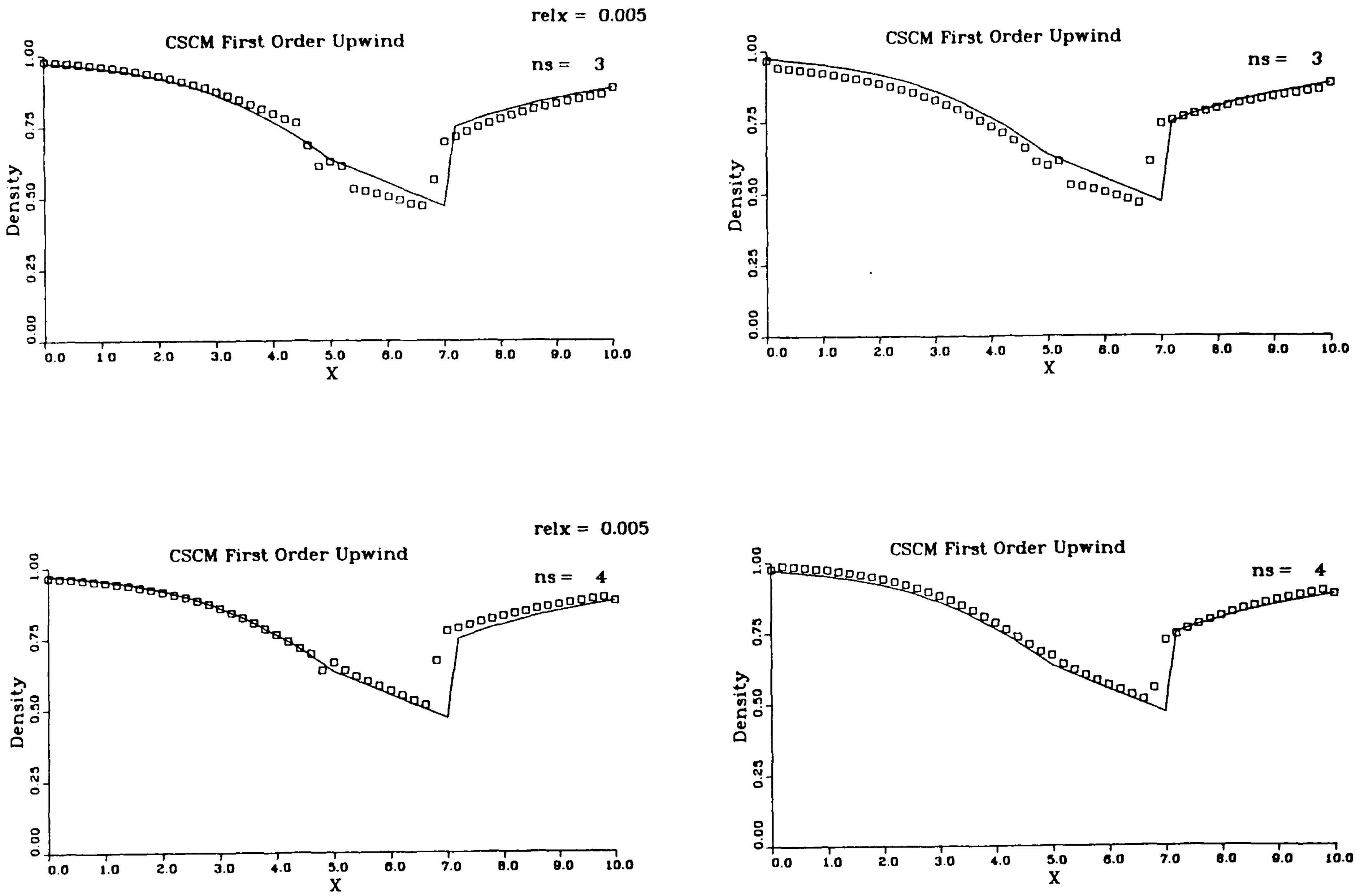


Figure 4. continued

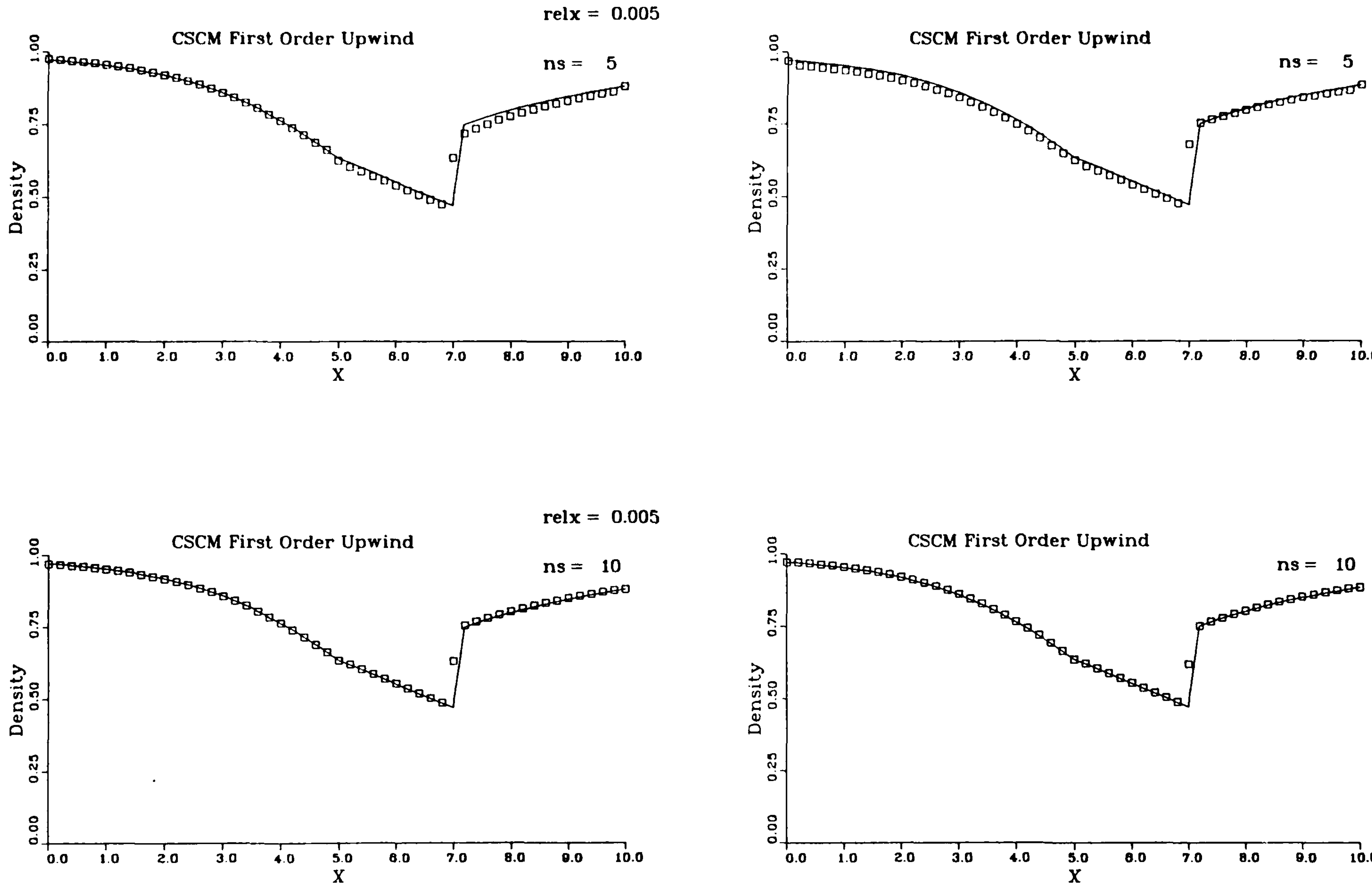


Figure 4. continued

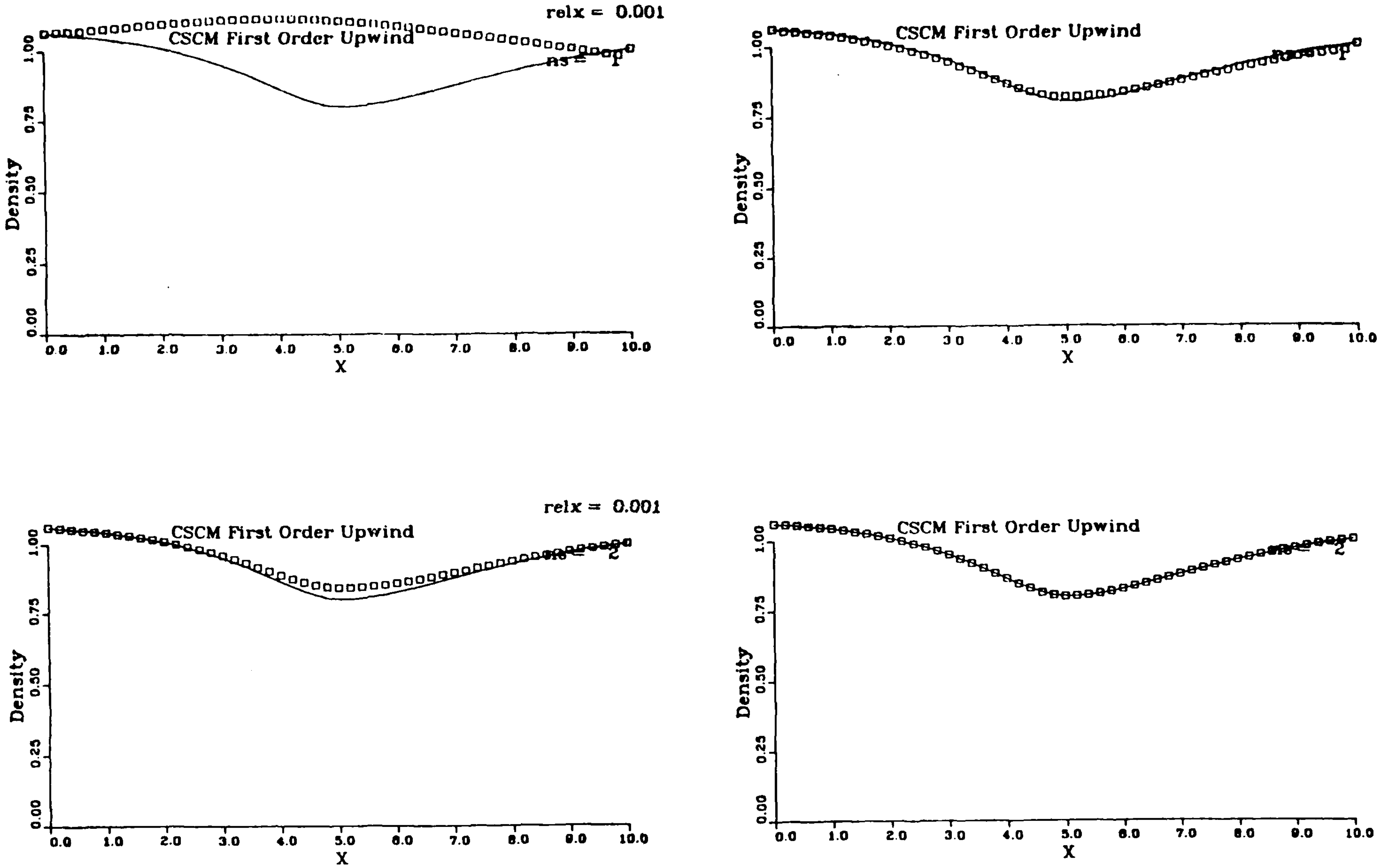


Figure 5. Subcritical solution to Blottner's converging-diverging nozzle developed with alternating sweeps in two global iteration steps. square - computed results , line - exact solution

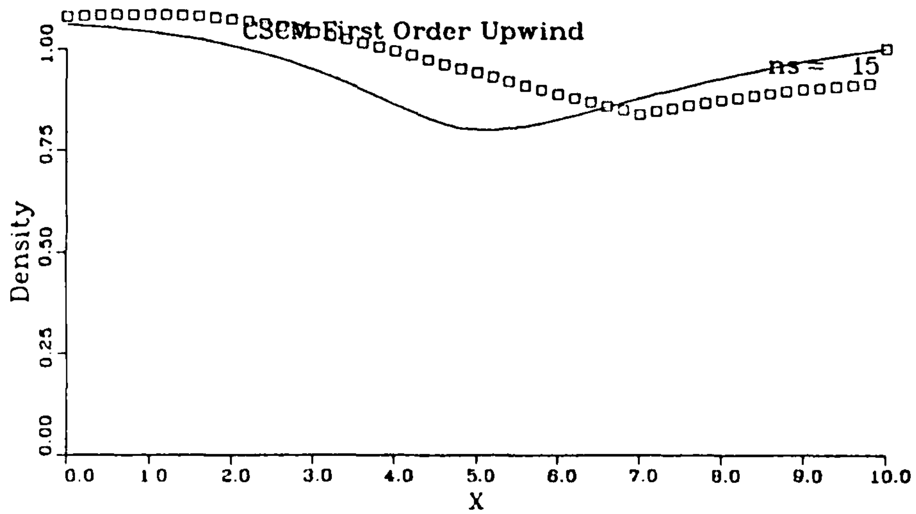


Figure 6. Solution to subcritical nozzle problem after 15 global iterations with forward marching sweeps only.

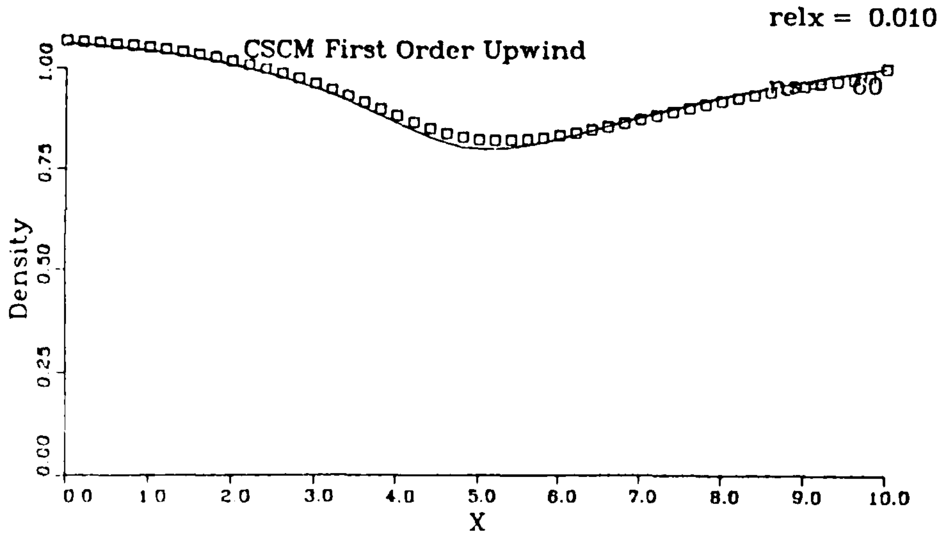


Figure 7. Solution to subcritical nozzle problem after 60 global iterations with forward marching sweeps only.

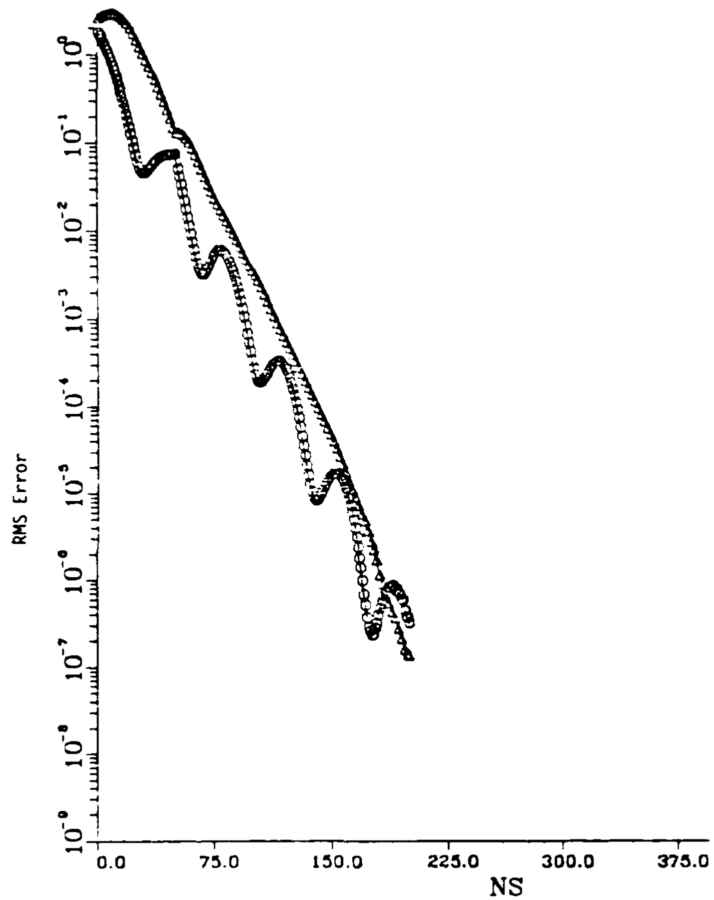


Figure 8. Convergence history of RMS error based on the exact steady solution for the subcritical nozzle flow solved with forward sweeps only.

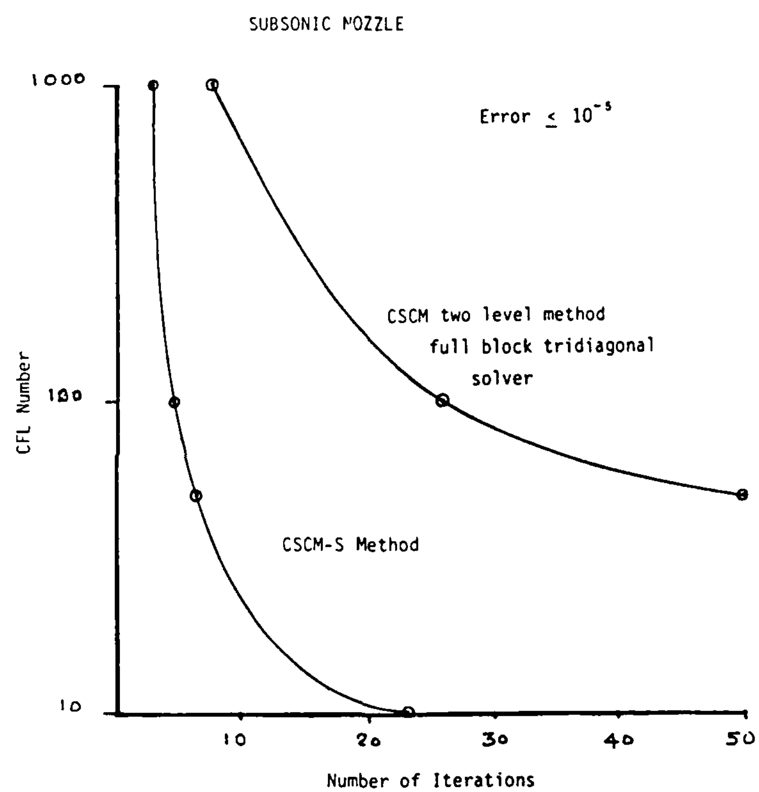


Figure 9. Convergence history comparison between the full block tridiagonal CSCM Solver and the CSCM-S method. The solutions have reached an RMS error less than 1×10^{-5}.

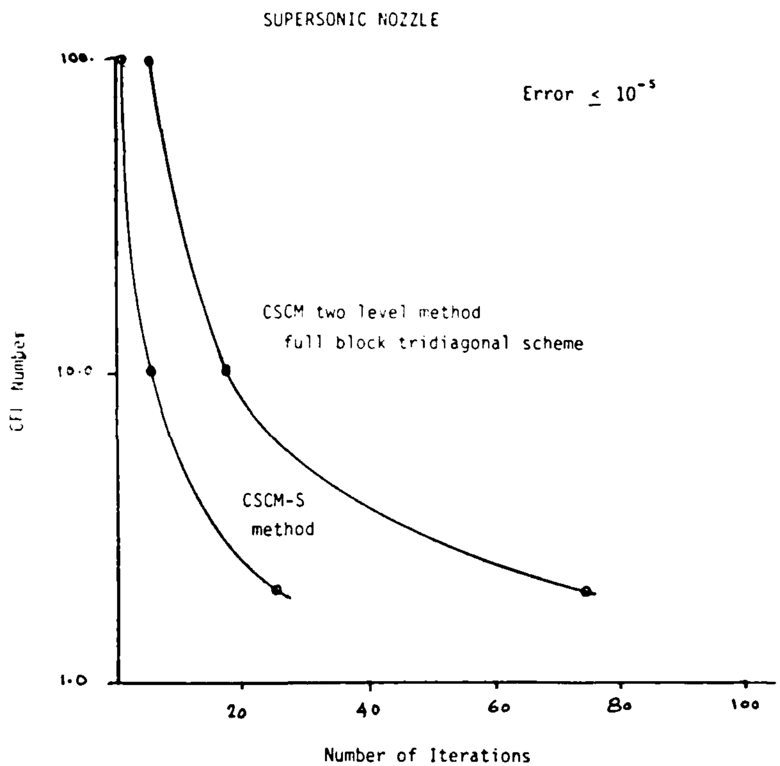


Figure 10. Convergence history comparison between the full block tridiagonal CSCM Solver and the CSCM-S method. The solutions have reached an RMS error less than 1×10^{-5}.

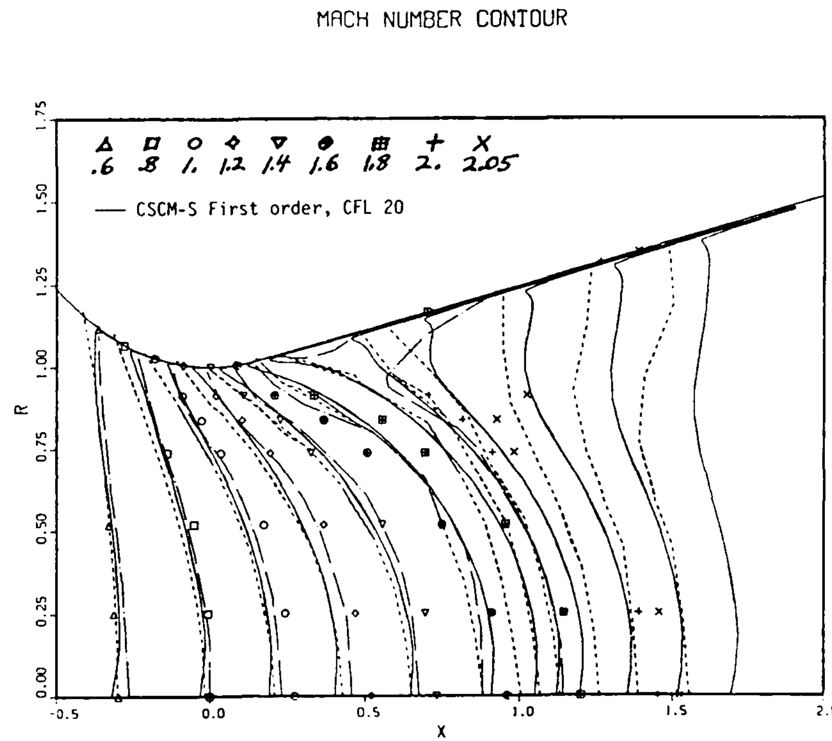


Figure 11. Mach number contours in vicinity of the throat for the axisymmetric transonic nozzle problem solved in ten global iterations. Other computed results of Cline (dashed line) and Prozan (chain dot) and experiment of Cuffel, et al (symbols) are discussed in references 22 and 28.

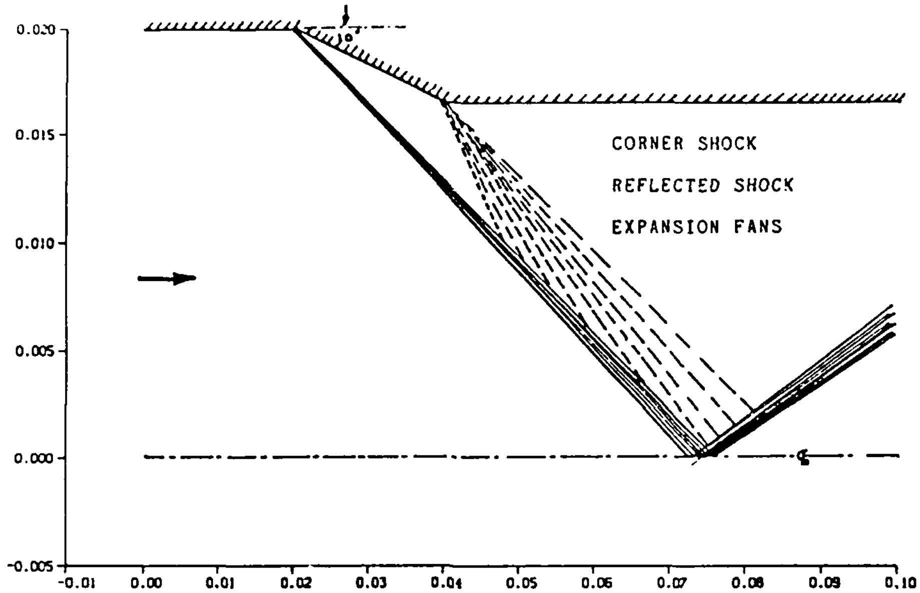


Figure 12. Schematics of the supersonic inlet flow problem.

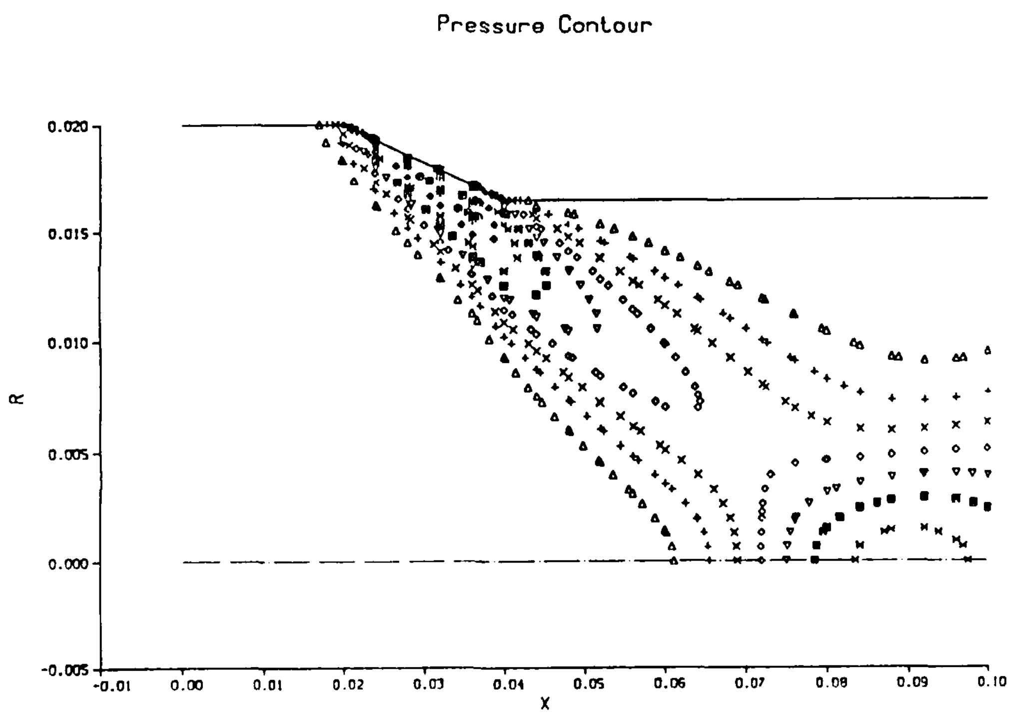


Figure 13. Pressure contours from the first order inviscid solution for the inlet problem after 10 forward sweeps with a 26 x 26 unstretched grid.

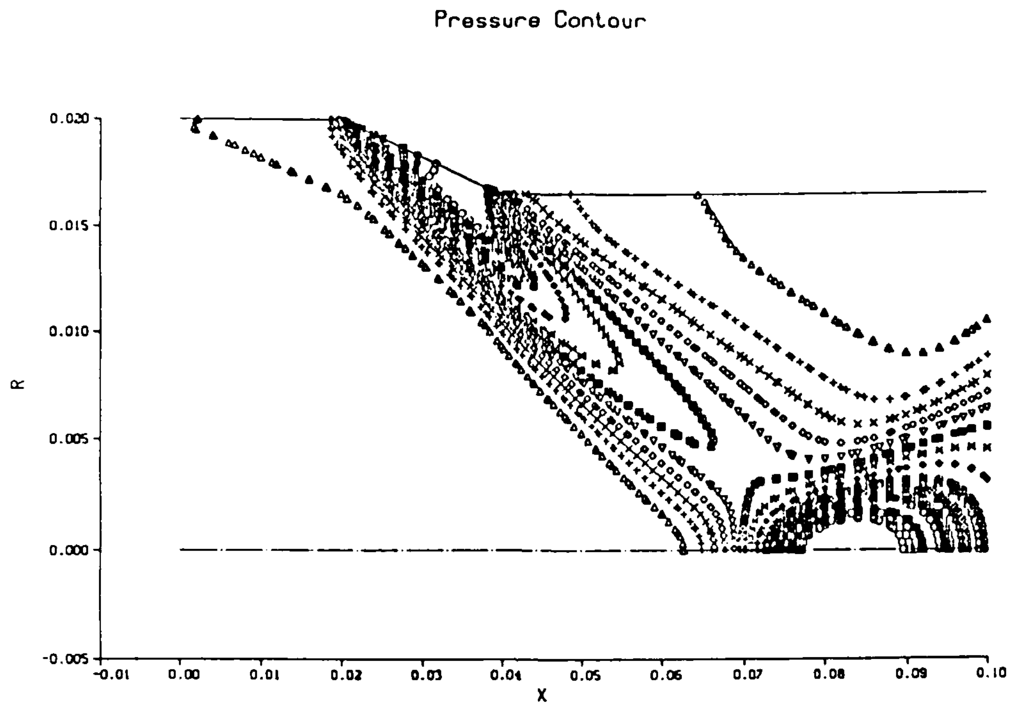


Figure 14. Pressure contours from the first order viscous solution for the inlet after 20 global sweeps with a 51 x 51 stretched mesh. Note the leading edge shock.

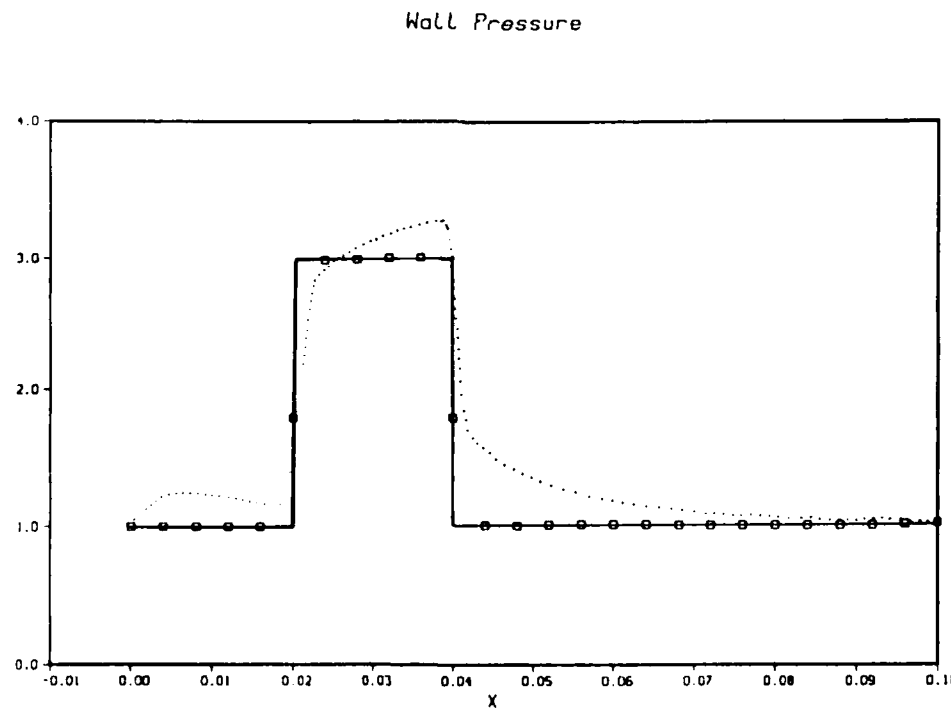


Figure 15. Wall pressure comparison. Line - exact inviscid solution, square - first order inviscid solution, dot - first order viscous solution.

RMS Residual History

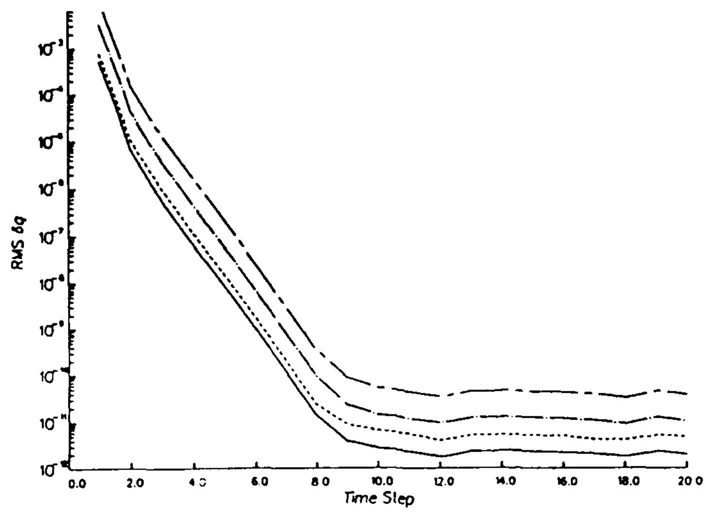


Figure 16. Convergence history of the RMS of the residuals for the inviscid first order inlet problem with 4 inner iterations per global sweep. Note at the end of the first sweep the residual has reduced and the solution has converged for all practical purposes.

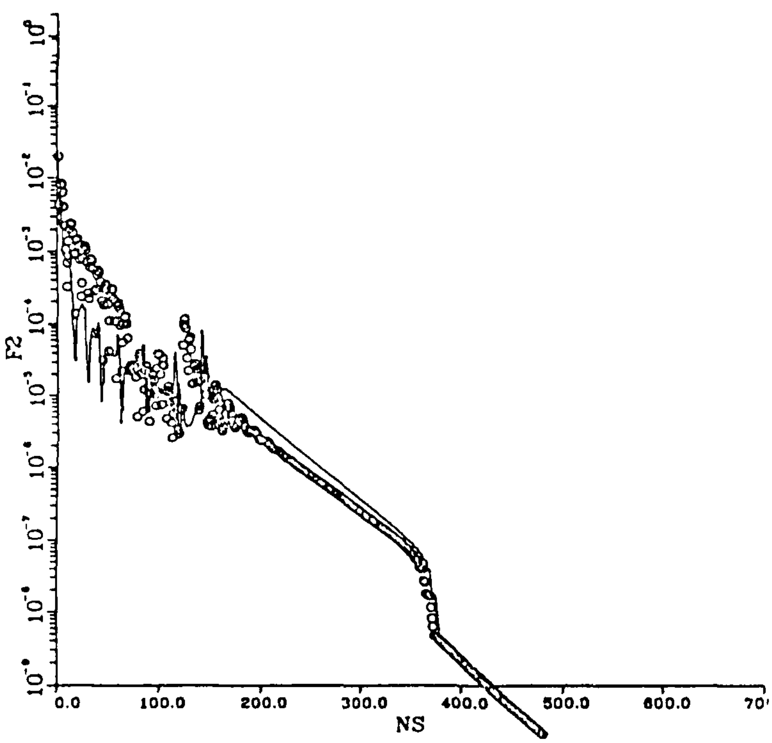


Figure 17. Test on three patches of implicit stability and rate of convergence of (circles) computed boundary point operator differencing to frozen data in adjacent patches; (solid line) effectively continuous grid method.

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